

State space models should be preferred to transfer function models in an early phase. You are allowed to initially introduce a few transfer function models but they should be converted to state space models as soon as possible!

1. Consider the effect of acceleration a to speed v and position p in linear motion ($a = \dot{v}$, $v = \dot{p}$). Assume the control law

$$a = -3 \cdot v - 2 \cdot p$$

- a) Calculate the eigenvalues of the closed-loop system.
- b) Calculate the phase margin and the delay margin.
- c) Simulate position, speed and acceleration for unity initial values of the velocity and position. 2,5
- d) Calculate $\int_0^\infty a^2(t) dt$ for unity initial values of the velocity and position. 8 p.

2. A valve includes a spring on the flapper to close the valve if power will be lost. The control input is the flapper torque. The state model of the system is below where the first state is the process output y to be controlled:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -10 & 1 \end{bmatrix} \cdot x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot u$$

Let the set point of the process output y be r . Design a control law of the form $u = K \cdot x + N \cdot r$ to make one of the closed-loop eigenvalues equal to $(-3 + 4 \cdot j)$ and to reach zero steady-state control error for a constant yet non-zero set point. 1,5
4 p.

3. Consider again the valve of *Problem 2* but now with the simpler control law $u = K \cdot x$. Let the performance index be

$$J = \int_0^\infty x_1^2(t) dt + 0.001 \cdot \int_0^\infty u^2(t) dt$$

Tune K to minimize J , and compute the minimum value of J when $x_1(0) = 1$, $x_2(0) = 0.1$. 0
4 p.

4. Consider again the valve of *Problems 2-3*. Design a state observer the eigenvalues of which are -10 , -10 . Your answer should also include a description of the observer either using equations or an informative block diagram 0
4 p.

5. A temperature sensor with unity gain and unity time constant is used to measure a temperature function which oscillates with sine form between the values 40 and 60 *Celsius* degrees with the time period of 10 seconds. Simulate the sensor output. 1
5 p.

6. The transfer function of a system is given below. Find a state space model for it and test if the model is controllable and/or observable. Also create a dual model and make similar tests to it. Finally, explain the observations by referring to key properties of the transfer function, properties found with suitable tools. 2,5
5 p.

$$G(s) = \frac{s+1}{s^2 + 3 \cdot s + 2}$$