ROBUST CONTROL WE 01

1. Calculate $\left\| \begin{bmatrix} 3 \\ 4 \cdot j \end{bmatrix} \right\|$ **1 p.**

2. The equilibrium (steady-state) model of a multivariable system is $y = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot u$. **3 p.**

Find the upper bound of $k = \dfrac{\| y \|}{\| u \|}$.

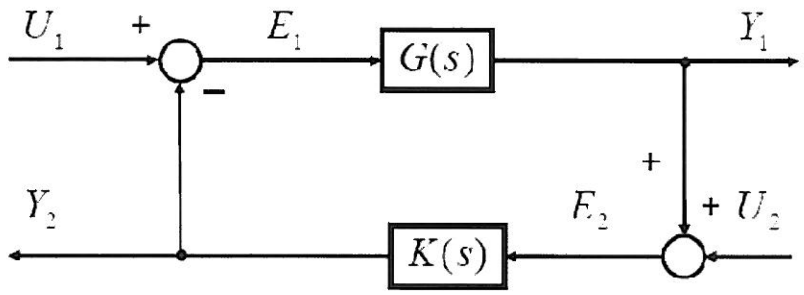
3. The input and the outputs of a **LTI** (*Linear Time-Invariant*) **MIMO** process oscillate with the basic period **T** so that they are linear combinations of a sine (sin) and a cosine (cos) function.The principal gains of the process can be computed by a software. Explain the use of the principal gains to explain something about the input-to-output relationship, including standard parameterizations of the oscillating functions. No non-trivial arithmetics needed. **2 p.**

ROBUST CONTROL WE 02

1. Consider a square system described by the **LTI** state model below. Assume that the model is both controllable and observable. Derive an equation satisfied by all the *zeros* of the system. **2 p.**

$$\dot{x} = A \cdot x + B \cdot u \ , \ y = C \cdot x + D \cdot u$$

2. Consider the system below. Derive a matrix suitable enough for studying internal stability. Any presentation of the matrix is acceptable (two good candidates were derived!). You can refer to facts which you remember or understand intuitively but mathematical calculus is needed so that writing the matrix directly is not enough. **4 p.**



ROBUST CONTROL WE03

1. A test matrix of internal stability for a system is **3 p.**

$$T = \left[\begin{array}{c|c} (I+K\cdot G)^{-1} & -(I+K\cdot G)^{-1}\cdot K \\ \hline (I+G\cdot K)^{-1}\cdot G & (I+G\cdot K)^{-1} \end{array}\right]$$

List a minimal set of conditions which together can reveal all the poles of the test matrix. What is the nature of the conditions, how should one proceed in the study given the conditions?

2. What is the original (primary) definition of $\| G \|_{\infty}$ for a **MIMO LTI** system ? **1 p.**

$\sup \bar{\sigma}(G(j\omega))$

3. How is $\| G \|_{\infty,\,[a\,,\,b]}$ defined for a finite time range $[a\ ,\ b\,]$? Why this norm is sometimes needed? Define also carefully objects which you need to mention. **2 p.**

$w \to [G] \to y$

$$\|G\|_{\infty\,[a,b]} = \frac{\|y(t)\|_{2,[a,b]}}{\|w(t)\|_{2,[a,b]}}$$

define 2-norm

$$\sqrt{\int_a^b w^T(t)\,w(t)\,dt}$$

ROBUST CONTROL WE04

1. Define including a figure: *Output Multiplicative Uncertainty* . **1 p.**

2. Which signal (function) norms were used to derive the theory of robust stability under unstructured uncertaintities? **1 p.**

3. The nominal plant model G and the controller model K are **4 p.**

$$G(s) = \frac{N_G(s)}{D_G(s)} \quad , \quad K(s) = \frac{N_K(s)}{D_K(s)}$$

where the objects above and below division lines are polynomials of s .
There is an unstructured input feedback uncertainty Δ_{fi} such that $\left\| \Delta_{fi} \right\|_\infty \leq a$.
One should study robust stability. A transfer function F and a *condition* associated with it can be used to perhaps find evidence on robust stability. Create a standard block diagram for the study , derive F and present the *condition.*

ROBUST CONTROL WE05

A **LTI** process with the state model $\dot{x} = A \cdot x + B \cdot u$ is controlled using the constant gain feedback control law $u = -K \cdot x$ where K has been chosen to minimize the performance index

$$J(K) = \int_0^\infty \left[x^T \cdot Q \cdot x + u^T \cdot R \cdot u \right] dt$$

It has been shown that for this control law we have the matrix identity

$$\left[I + K(j\omega \cdot I - A)^{-1} B \right]^+ \cdot R \cdot \left[I + K(j\omega \cdot I - A)^{-1} B \right] = R + \left[(j\omega \cdot I - A)^{-1} B \right]^+ \cdot Q \cdot (j\omega \cdot I - A)^{-1} B$$

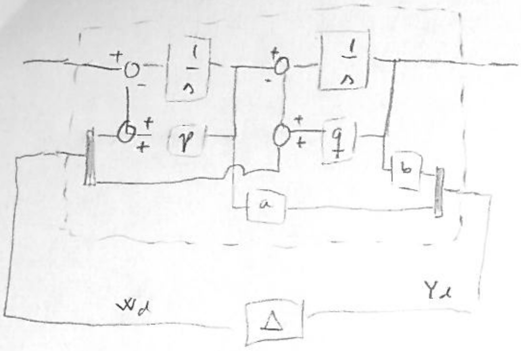
for all real values of ω .

a) Derive a tight lower bound for the phase margin for single input processes. **4 p.**

b) For multi-input systems there exists a nice sufficient condition for robust stability for a certain uncertainty type under a certain condition on J. Which uncertainty type , which condition? **2 p.**

input multip ehdo painot ovat $\rho I = R$ eli verrannollisia identiteettiin

ROBUST CONTROL WE06



$\det(I - \frac{Y_d}{W_d}\Delta) \neq 0$
$\Rightarrow$ stabiili

1. A system with the model structure

$$G(s) = \frac{1}{(s+p)\cdot(s+q)}$$

should be controlled . The process parameters are know to satisfy the "box" conditions

$$p_{\min} \le p \le p_{\max} \;,\; q_{\min} \le q \le q_{\max}$$

Present a standard block diagram for a concluding the existence or lack of robust stability. **2 p.**

2. In an application a normalized perturbation $\Delta(s)$ may imply poles which are the solutions of the equation

$$\det(I + M(s)\cdot\Delta(s)) = 0$$

where $M(s)$ is a known matrix. Define a *structured singular value* for a stability study and give an inequality condition to its value. **2 p.**

$$\mu_\Delta = \frac{1}{\min_{\Delta\in\bar{\Delta}}(\bar{\sigma}(\Delta) \text{ s.e. } \det(I - N\Delta) = 0)}$$

$\mu_\Delta = 0$ jos $\det \neq 0 \; \forall \Delta$

3. Give an easy-to-get upper bound and a easy-to-get lower bound for the structured singular value. The bounds requested here are allowed to be conservative in some cases. **2 p.**

upper $\mu_\Delta(N) \le \bar{\sigma}(N)$

lower $\mu_\Delta(N) \ge \rho(N)$, $\rho(N) = \max_i |\lambda_i(N)|$

ROBUST CONTROL WE06 Release 2.0

1. In an application a normalized perturbation $\Delta(s)$ may imply poles which are the solutions of the equation

$$\det(I + N(s) \cdot \Delta(s)) = 0$$

where $N(s)$ is a known matrix. Define a *structured singular value* for a stability study and give an inequality condition to its value. **2 p.**

2. The *structured singular value* may be found an upper bound using so called $\mathcal{D}$-scales. Explain how in many enough essential details! **2 p.**

3. The *structured singular value* may be found an lower bound through certain eigenvalue calculus. Explain the technique and its weaknesses with modest mathematical presentation and mostly with words! **2 p.**

ROBUST CONTROL WE07

Write a compact story about general enough specifications of robust performance , and present a systematic procedure for studying if the specifications are achieved. Include many enough informative enough block diagrams assuming state feedback control augmented with suitable additional features. However, you are not supposed to derive precise mathematical models needed in the ultimate Matlab tests. Regard yourself as an instructor of research/project work, i.e. that you are explaining the ideas for a less experienced student/colleague.

ROBUST CONTROL WE08

The topic is associated with certain optimal disturbance attenuation for a LTI process.

1. Some scalar functions have a saddle point. What/which kind a point is it? **1 p.**

2. Present a *normalized LTI model*, the *performance index* (for a suitable vector associated with process and controller variables) with *motivation* of the structure, and a description of a sub-optimal strategy to optimize the index.

Here you are not supposed to 1) work with Hamiltonians 2) present computational details of the algorithms used typically in optimization. **3 p.**

3. Sometimes the optimization problem defined does not have a sub-optimal solution.

a) Under which condition the solution searched does not exist? **1 p.**

b) There exists an eigenvalue test to study if the solution exists. Describe its nature in words. **1 p.**

ROBUST CONTROL WE09

1. The linear time-variant model (1ab) below can be defined an adjoint model. Define it and tell how the impulse response (matrix) of the adjoint model depends on that of model (1) .

$$\dot{x}(t) = A(t) \cdot x(t) + B(t) \cdot u(t) \quad (1a)$$

$$y(t) = C(t) \cdot x(t) + D(t) \cdot u(t) \quad (1b)$$

2 p.

2. Tell briefly about estimation associated with **Linear Time-Invariant** state space system:

a) about the need of estimation , assumptions made and modelling conventions of **Burl** to pose the problem

b) the performance index to be minimized

c) the clever trick mentioned in **Burl** used to derive computational procedures

4 p.

ROBUST CONTROL WE10

1. A **MIMO** state space process has been designed **LTI** state feedback:

$$\dot{x} = A \cdot x + B \cdot u$$

$$u = -K \cdot x$$

However, the controller will be moved to far away from the process so that the control becomes remote control through a telecommunication network where the signal processing delay between the process room and control room is guaranteed to be the constant d . How can you check if re-design of K is needed to maintain closed-loop stability? Here also mathematical statements, derivations, descriptions as well as some calculus are needed! **2 p.**

2. In the transfer function

$$G(s) = \frac{a}{s} + \frac{c \cdot (s - z_1)}{(s - p_1) \cdot (s - p_2)}$$

with real parameters we have

$$z_1 \approx p_1 << p_2 < 0 \quad , \quad a \approx 0$$

Suggests a reasonable lower order approximation to G (you have certain freedom here!). **2 p.**

3. For process tfm (transfer function matrix) G , ideal measurement system , controller tfm F and controller tfm perturbation ΔF the perturbation of the closed loop tfm T is **2 p.**

$$\Delta T = (I + G \cdot F)^{-1} \cdot G \cdot \Delta F \cdot (I + G \cdot (F + \Delta F))^{-1} \approx (I + G \cdot F)^{-1} \cdot G \cdot \Delta F \cdot (I + G \cdot F)^{-1}$$

Derive an upper bound for the approximation of $\| \Delta T \|_\infty$ due to the approximation shown above.