AUT.710 Fundamentals of Mobile Robots

-Midterm check part 2-

Date: 24.4.2024

Instructions:

1. **Write your name and student ID on each of your answer sheets.**
2. The midterm check consists of 3 problems.
3. Describe how you arrive at the final solution and be clear. For example, write clearly the sign of the controller gain (i.e., whether $k > 0$ or $k < 0$), write the range of the orientation error, etc.
4. This is an individual test and thus no collaboration is allowed.

Problem 1 (3 points). Consider a unicycle/differential drive robot whose kinematic model is given by

$$\dot{x} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} u, \text{ where } x = \begin{bmatrix} p_x \\ p_y \\ \theta \end{bmatrix}, \; u = \begin{bmatrix} v \\ w \end{bmatrix}. \tag{1}$$

(i) Design a feedback control input u to make the robot position $\begin{bmatrix} p_x \\ p_y \end{bmatrix}$ converge with **zero error** to the goal position $g = \begin{bmatrix} a \\ b \end{bmatrix}$ as shown in Figure 1. **Note**: robot's orientation at the goal position can take any values. (2 points)

(ii) When implementing the control input u on a real robot, can we set the controller gains to any values? explain your answer (0.5 points)

(iii) Assume constant/fixed controller gains. Discuss how the ratio of the controller gains should be chosen for ensuring convergence to the goal position. (0.5 points)

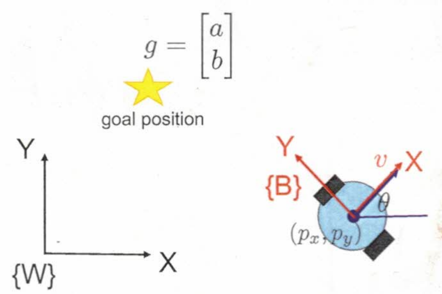

Figure 1: Scenario for Problem 1

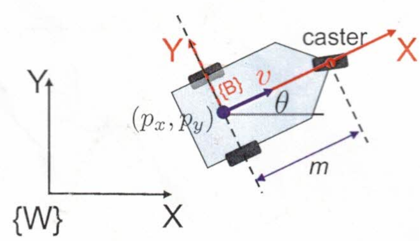

Figure 2: Scenario for Problem 2

Problem 2 (3.5 points). Consider a unicycle robot with a caster as shown in Figure 2 whose kinematic model is given by (1).

(i) Design control input u to make the caster track the trajectory $d(t) = \begin{bmatrix} d_x(t) \\ d_y(t) \end{bmatrix}$ (defined in world frame coordinate) with **zero error**. (1.5 points)

(ii) Assume that we modify the position of the caster along the X-axis of the robot's body frame by decreasing the value of m. How does it affect control input $u = [v, \omega]^T$? (0.8 points)

(iii) Using the controller designed in (i) of Problem 2, is it possible to make the robot's position $[p_x, p_y]^T$ track trajectory $d(t)$ with **zero error** by decreasing the value of m to zero? Explain your answer (0.7 points)

(iv) If the radius of the wheel is equal to r and the distance between the wheels is equal to D, compute the angular velocity of the right and left wheels, i.e., w_r, w_l for the control input computed in (i). (0.5 points)

Problem 3 (3.5 points). Consider an **omnidirectional** robot with radius $0.2m$ whose kinematic model (by neglecting its orientation) is given by $\dot{x} = u$ with $x = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$ and $u = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$. We aim to design u using QP-method to make the robot reach its goal position x^d while avoiding two static obstacles o_1 and o_2 with radius $R_{o_1} = 0.3m$, $R_{o_2} = 0.4m$ and located at $x_{o_1} = [-0.6, 0]^T$ and $x_{o_2} = [0.6, 0]^T$ as illustrated in Figure 3. To this end, we formulate the following QP:

$$\begin{aligned} \min_u \quad & \|u_{gtg} - u\|^2 \\ \text{s.t.} \quad & \dot{h}_{o_1}(x, u) \geq -\gamma_o h_{o_1}(x), \\ & \dot{h}_{o_2}(x, u) \geq -\gamma_o h_{o_2}(x), \; \gamma_o > 0 \end{aligned} \quad \Longleftrightarrow \quad \begin{aligned} \min_u \quad & \frac{1}{2} u^T Q u + c^T u \\ \text{s.t.} \quad & Hu \preceq b \end{aligned} \tag{2}$$

where u_{gtg} is the go-to-goal controller designed without considering the obstacles, $h_{o_1}(x) \geq 0$ and $h_{o_2}(x) \geq 0$ denote that the states are within the safe set (i.e., no collision with each static obstacle). Assuming that the robot knows the obstacles location and size and there is no actuator constraints, answer the following questions.

(i) Write the expressions of $h_{o_1}(x)$ and $h_{o_2}(x)$ and how their parameter values should be chosen to ensure no collisions with the obstacles. (1 point)

(ii) Moreover, compute the matrix H and vector b in (2) together with their dimensions. (1 point)

(iii) To make the robot move **between the two obstacles** for reaching the goal while avoiding collision, describe how to choose the value of parameter γ_o of the QP-based safety control in (2) **and** the range of the parameter values (i.e., safety distance) for $h_{o_1}(x)$ and $h_{o_2}(x)$. (1.5 points)

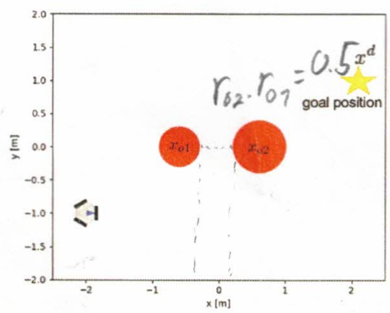

Figure 3: Omnidirectional robot with two static obstacles