MATH.APP.220 Multivariable Calculus / Hirvonen

Exam 15.05.2024

No calculators, no written material. A collection of formulas is on the flipside.

1. Consider the function $f(x, y) = 2x\sqrt{x + 2y}$.

   (a) Find the directional derivative of f at $(4, 6)$ in the direction $(1, -2)$.

   (b) Find a direction in which the value of f doesn't change at $(4, 6)$.

   (c) Find the direction where f decreases the most at $(-2, 3)$.

2. Find the extreme values of $f(x, y) = x^3 + y^2 - 3x^2 + 6y + 6$ in a triangle bounded by the lines $x = 1$, $y = -4$ and $y = 2x - 2$.

3. Consider the curves C_1 and C_2 with parametrizations

$$C_1: \quad \mathbf{r}_1(s) = (s - 2, 2s - 5), \quad s \in [0, 3],$$
$$C_2: \quad \mathbf{r}_2(t) = (2\cos t, 2\sin t), \quad t \in [0, 2\pi].$$

   (a) Represent the curves C_1 and C_2 using only the variables x and y. Remember to give the domain of x.

   (b) Out of the following three points $(0, 4)$, $(-2, 0)$, $(2, -2)$, denote by $\mathbf{a}$ the only point that is on the curve C_2. If $\mathbf{r}_2$ is the location of a center of mass at the instant t, what is the acceleration of the center of mass at the point $\mathbf{a}$?
   Hint: After deciding which point is $\mathbf{a}$, find the instant t when the center of mass is located at the point $\mathbf{a}$.

4. Use a triple integral to calculate the volume of the object bounded by the surfaces $z = 1$, $y = 0$ and $z = x^2 + y^2$.

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Exam Formula Sheet

1. $T(\mathbf{x}) = F(\mathbf{a}) + F'(\mathbf{a})(\mathbf{x}-\mathbf{a})$

2. $(F \circ G)'(\mathbf{x}) = F'(G(\mathbf{x}))\, G'(\mathbf{x})$

3. $$F'(\mathbf{x}) = \begin{bmatrix} D_1 f_1(\mathbf{x}) & D_2 f_1(\mathbf{x}) & \cdots & D_n f_1(\mathbf{x}) \\ D_1 f_2(\mathbf{x}) & D_2 f_2(\mathbf{x}) & \cdots & D_n f_2(\mathbf{x}) \\ \vdots & \vdots & \ddots & \vdots \\ D_1 f_m(\mathbf{x}) & D_2 f_m(\mathbf{x}) & \cdots & D_n f_m(\mathbf{x}) \end{bmatrix}$$

4. $D_{\mathbf{e}} f(\mathbf{x}) = \nabla f(\mathbf{x}) \bullet \mathbf{e}$

5. $$\iint_R f(x,y)\, dxdy = \int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, drd\theta$$

6. $$\begin{cases} x = \rho\sin\phi\cos\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases} \qquad dxdydz = \rho^2 \sin\phi\, d\rho d\phi d\theta$$

7. $$m = \iiint_T \rho(x,y,z)\, dV$$
$$\overline{x} = \frac{1}{m}\iiint_T x\rho(x,y,z)\, dV, \quad \overline{y} = \frac{1}{m}\iiint_T y\rho(x,y,z)\, dV, \quad \overline{z} = \frac{1}{m}\iiint_T z\rho(x,y,z)\, dV$$
$$I_z = \iiint_T (x^2+y^2)\,\rho(x,y,z)\, dV$$

8. $$\iint_A f(x,y)\, dxdy = \iint_S f(F(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv$$

9. $$\int_a^b f'(g(x))\, g'(x)\, dx = \Big[f(g(x))\Big]_a^b$$
$$\int_a^b f'(x)\, g(x)\, dx = \Big[f(x)\, g(x)\Big]_a^b - \int_a^b f(x)\, g'(x)\, dx$$
$$\int_a^b \frac{f'(x)}{f(x)} dx = \Big[\ln|f(x)|\Big]_a^b$$

10. $\sin^2 t = \frac{1}{2}(1-\cos(2t)), \qquad \cos^2 t = \frac{1}{2}(1+\cos(2t))$