SGN-2206 Adaptive Signal Processing

Examination March 2010

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Calculators of all kinds can be used in the exam.

Those not having a calculator can leave the result of arithmetic operations unevaluated (but the operations to be performed should be written down completely, using intermediate notations if needed)

1. (4 points) State the problem of optimal filter design for the backward predictor (model, data available, criterion to be minimized).

2. (3 points) (a) Find the least squares estimate of w_0 in the very simple model $y(t) = w_0$ when the desired data $d(t)$ is given for $t = 1, \ldots, N$ (therefore the input is assumed $u(t) = 1$ for $t = 1, \ldots, N$). What is the significance of the estimate?

   (3 points) (b) Find the recursive least squares solution $w_0(N)$ for the model at (b) as a simple equation connecting $w_0(N)$ to $w_0(N-1)$ (by elementary derivations, no need to use the general RLS equations). Try to find also the exponentially weighted solution ($\beta(n,i) = \lambda^{n-i}$).

3. (6 points) Consider a FIR(1) filter $y(n) = w(n)u(n)$ where all quantities are scalars. We intend to minimize the time varying cost function

$$J(n) = e(n)^2 + \alpha w(n)^2$$

where $e(n)$ is the estimation error

$$e(n) = d(n) - w(n)u(n)$$

$d(n)$ is the desired response, $u(n)$ is the input, and α is a constant. Show that the time update for the parameter vector $w(n)$ is defined by

$$w(n+1) = (1 - \mu\alpha)w(n) + \mu u(n)e(n)$$

What is the role of the constant α (comment the cases of very large α and very small α).

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4. • (4 points) The normalized LMS (NLMS) algorithm can be derived as a constrained optimization algorithm for fulfilling the equality $\underline{w}(n)^T \underline{u}(n) = d(n)$. Show how you can apply the lagrange multiplier method to derive the NLMS algorithm.

   • (2 points) Write the normalized LMS algorithm for the FIR filter with two paramteres, w_0 and w_1.

   How the algorithm will evolve if the input is $u(0) = 0, u(1) = 0, u(2) = 1, u(3) = 1, u(4) = u(5) = u(6) = ... = 0$ and the desired input is $d(0) = 0, d(1) = 0, d(2) = 0, d(3) = 1, d(4) = d(5) = d(6) = ... = 0$ (consider different situations for the initial weights).

5. (4 points) Consider a sigmoidal perceptron. Write its model, the training equations and the diagram showing the flow of computations.

6. (4 points) Draw the structure of an adaptive echo canceler. Discuss the significance of each signal.