Final Exam 8.4.2010

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Only the calculator of the faculty may be used in the exam.

- 1. Let $a \neq b$. Which of the following signals are root signals of the median filter of length 3?
 - a) ..., a, a, \dots (1 point)
 - b)...,a,a,a,a,a,a,b,b,a,a,a,a,a,... (1 point)
 - c) ..., $a, a, a, a, a, a, a, b, b, b, b, b, b, b, \dots$ (1 point)
 - d)...,a,b,a,b,a,b,a,b,a,b,... (1 point)
 - e) ..., a, a, b, b, a, a, b, b, a, a, b, b... (1 point)
 - f) ..., a, a, a, b, b, b, a, a, a, b, b, b ... (1 point)
- 2. Consider the 6×6 image

7	6	10	6	1	5
4	4	6	3	5	3
0	7	7	5	5	4
2	0	2	0	4	3
7	4	3	3	1	0
2	2	3	2	6	3

window				
x_1	X_2	x_3		
x_4	X_5	x_6		
x_7	x_8	X_9		

What is the output image of recursive stack filter of size 3×3 and defined by the positive Boolean function $f(\mathbf{x}) = x_4 x_5 x_6 + x_2 x_5 x_8 + x_1 x_3 x_5 x_7 x_9$. No appending is required, but specify the scanning order that you have used. (6 points)

- 3. Does the Hodges-Lehmann D-filter coincide with the Wilcoxon filter when:
 - a) N = 3, (3 points)
 - b) N = 7. (3 points)

Justify your answers.

4. Assume that $X^* = X_{k+1}$ and m is the sample mean. Write the definition

LLMMSE
$$(X_1, X_2, ..., X_N) = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2} X^* + \left(1 - \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2}\right) m$$

in the form of

$$b(X_1 + X_2 + \ldots + X_k + aX_{k+1} + X_{k+2} + \ldots + X_N) \,.$$

This indicates that the LLMMSE filter is a weighted sum, with the center sample weighted differently from the others. When is the center sample given more weight than the other samples? (6 points)

- 5. Below is a part of the familiar test image used on the course and filtered versions of it. Give the corresponding output I1-I6 for each filter a-f. Size of the images is 85×165 and the filter windows are 5×5 .
 - a) 10/25 -trimmed mean filter (1 point)
 - b) K-nearest neighbor filter (K = 9) (1 point)
 - c) ranked-order filter (r = 19) (1 point)
 - d) R1LH+ filter (1 point)
 - e) harmonic mean filter (1 point)
 - f) recursive median filter (1 point)

