# 8005535 Signal Compression
# Examination on February 2001

1. (2 point) How one can encode a binary tree? (this is needed e.g. for sending the information on which prefix code was used for coding) Use an example of tree with maximum depth equal to 3.

2. (3 point) Which is the length of the Shanon's code for a symbol $j$ having probability $p_j$. Specify the Shanon code for the three symbol alphabet, having probabilities $p_1 = 0.51, p_2 = 0.26, p_3 = 0.24$.

3. (3 point) Show that by using a prefix code for blocks of symbols, (instead of a prefix code for individual symbols) the codelength is closer (or equal) to the entropy.

4. (4 point) You are given a Bernoulli source with $p = 0.6$ and asked to design a Tunstall tree with 8 leaves. What is the rate of the code?

5. (3 point) What predictor is used in JPEG-LS? What is the residual distribution assumed in JPEG-LS?

6. (2 point) How efficient is an arithmetic coder in encoding the bitstream produced by SPIHT method?

7. (5 points) a) Find the binary Huffman code for the source with probabilities $\left(\frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{2}{15}, \frac{2}{15}\right)$. Argue that this code is also optimal for the source with probabilities $\left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)$.

   b) Consider a random variable $X$ which takes on four values with probabilities $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{4}, \frac{1}{12}\right)$.

   - Construct a Huffman code for this random variable.
   - Show that there exists two different sets of optimal lengths for the codewords, namely, show that codeword length assignments $(1, 2, 3, 3)$ and $(2, 2, 2, 2)$ are both optimal. Compare the lengths of the codewords corresponding to Huffman coding with the lengths corresponding to Shannon coding.

8. (3 points)

   Which of these codes cannot be Huffman codes for any probability assignment ?

   a) $\{0, 10, 11\}$

   b) $\{00, 01, 10, 110\}$

   c) $\{01, 10\}$

9. (5 points)

   Many practical coders use the Golomb($\ell$) codes in the particular hypothesis that $\ell$ is a power of 2. Generate the codewords and the trees for $\ell = 7$ and $\ell = 8$ and explain the implementation advantages when $\ell = 8$. Derive the code length when the unsigned integer $u$ is encoded with Golomb codes for which $\ell = 2^k, k \in \mathcal{Z}$.